

Spontaneous Compactification of Bimetric Theory

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Abstract

We propose a model of bimetric gravity in which the mixing of metrics naturally gives a mass to a graviton by the compactification with flux of two gauge fields in extra dimensions. We assume that each metric in the solution for the background geometry describes the four-dimensional Minkowski spacetime with an S^2 extra space, though the two radii of S^2 for two metrics take different values in general. The solution is derived by the effective potential method in the presence of the magnetic fluxes on the extra spheres. We find that a massive graviton is governed by the Fierz-Pauli Lagrangian in the weak field limit and one massless graviton left in four dimensions.

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1 Introduction

In the last ten years or so, there have been a number of attempts to understand the mysterious components in the universe, called as dark matter and dark energy. One of the possibilities is to consider the modifications (for reviews, [1, 2]) of the Einstein gravity, which has been the theoretical basis for analyzing the universe as a whole. We have become aware of ignorance of the nature of

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gravitational interaction at large distances and at very small distance. This fact inspires a search for modifications of the general relativity at large and small distances. Recently, many authors study massive graviton theory (for reviews, [3, 4]) with much interest for understanding cosmology and other aspects of gravity. The progress to the consistent massive theory has been attained by introducing another (to be non-dynamical) metric in the theory [5, 6]. Besides them, the bigravity theory or bimetric theory of gravity has a long history since 1970's [7, 8], which describes massless and massive gravitons in general. In the massive theories, the mass of the graviton is given by hand. The lack of theoretical explanation of the origin of the mass scale is a shortcoming of the generic massive gravity models.

The mixing term of two metrics gives a mass to a graviton. In the present paper, we consider a model in which the mixing originates from the fluxes of gauge fields in extra dimensions.

A model of multiple gauge fields with a kinetic mixing has been considered three decades ago [9] and studies on possible consequences in similar models have repeatedly appeared until recent times [10, 11, 12]. A possible stringy origin of such a type of models has also been investigated [13, 14]. In their models, the Lagrangian for two gauge fields is written as in the form:

$$\mathcal{L} = -\frac{1}{4} (F_{1\mu\nu}F_1^{\mu\nu} + F_{2\mu\nu}F_2^{\mu\nu} + 2\alpha F_{1\mu\nu}F_2^{\mu\nu}) , \quad (1)$$

where α is a *dimensionless* constant. The models suggest existence of exotic particles and candidates of dark matter. Being motivated partially by the work, we come to an idea of using two gauge fields as well as two metrics in the theory. The mixing term of two gauge kinetic part can play a role of mixing of two metrics simultaneously.

We consider the simplest compactification in a six-dimensional model with $U(1)$ gauge field strengths in the extra space. This yields the non-derivative interaction of two metrics in the vierbein formalism [15, 16, 17, 18, 19]. In the present paper, we analyze our model as the on-shell level, and quantum nature in the model is not discussed here.

In the next section, we define our model. In Sec. 3, after assumptions for the metrics with compactification and magnetic fluxes in the extra space are declared, compactification with four-dimensional Minkowski spacetime is investigated by means of the effective potential. In Sec. 4, the effective four-dimensional Lagrangian for gravitons in the weak field limit is derived. Finally, we summarize our work and give remarks about the outlook in Sec. 5.

2 The model

Our model has two metrics g_{MN} and f_{MN} , and two $U(1)$ gauge fields A_{gM} and A_{fM} , where M, N run over $0, 1, 2, 3, 5$, and 6 . The field strengths are defined usually as $F_{gMN} = \partial_M A_{gN} - \partial_N A_{gM}$ and $F_{fMN} = \partial_M A_{fN} - \partial_N A_{fM}$. We

consider the following action for six dimensional spacetime expressed as

$$S = S_g[g, F_g] + S_f[f, F_f] + S_{int}[g, f, F_g, F_f], \quad (2)$$

where

$$S_g = \int d^6x \sqrt{-g} \left[\frac{1}{2\kappa_g^2} R_g - \frac{1}{4} g^{MK} g^{NL} F_{gMN} F_{gKL} - \Lambda_g \right] \quad (3)$$

and

$$S_f = \int d^6x \sqrt{-f} \left[\frac{1}{2\kappa_f^2} R_f - \frac{1}{4} f^{MK} f^{NL} F_{fMN} F_{fKL} - \Lambda_f \right]. \quad (4)$$

Here R_g and R_f are the Ricci scalars constructed from the metric g and f , respectively. The quantities denoted by $\kappa_g, \kappa_f, \Lambda_g, \Lambda_f$ are constants.

To analyze the preferred action S_{int} including the mixing term, we introduce two sechbeins e_g and e_f which satisfy

$$e_{gM}^A \eta_{AB} e_{gN}^B = g_{MN}, \quad e_{fM}^A \eta_{AB} e_{fN}^B = f_{MN}, \quad (5)$$

where $\eta_{AB} = \eta^{AB} = \text{diag.}(-1, 1, 1, 1, 1, 1)$. Now we adopt the following action S_{int} , written with help of antisymmetric symbols:

$$S_{int} = -\frac{\alpha}{96} \int d^6x \epsilon^{MNRSTL} \epsilon_{ABCDEF} e_{gM}^A e_{fN}^B e_{gR}^C e_{fS}^D F_{gTL} F_{fJK} e_g^{EJ} e_f^{FK}, \quad (6)$$

where $\eta_{AB} e_{g(f)}^{AM} e_{g(f)N}^B = \delta_N^M$. The *dimensionless* coupling constant α satisfies $|\alpha| < 1$. Note that this term has two reflection symmetries $e_g \leftrightarrow -e_g$ and $e_f \leftrightarrow -e_f$ independently, and an exchange symmetry $e_g \leftrightarrow e_f$. Though this action seems a bit bizarre, the other term in $S_{g(f)}$ can also be rewritten using sechbeins and one can see

$$\sqrt{-g} (F_g)^2 = \frac{1}{48} \epsilon^{MNRSTL} \epsilon_{ABCDEF} e_{gM}^A e_{gN}^B e_{gR}^C e_{gS}^D F_{gTL} F_{gJK} e_g^{EJ} e_g^{FK}, \quad (7)$$

$$\sqrt{-g} = \det e_g = \frac{1}{720} \epsilon^{MNRSTL} \epsilon_{ABCDEF} e_{gM}^A e_{gN}^B e_{gR}^C e_{gS}^D e_{gT}^E e_{gL}^F \quad (8)$$

and

$$\sqrt{-g} R_g = \frac{1}{48} \epsilon^{MNRSTL} \epsilon_{ABCDEF} e_{gM}^A e_{gN}^B e_{gR}^C e_{gS}^D R_g^{EF}{}_{TL}, \quad (9)$$

where $R_g^{EF}{}_{TL}$ is identical to the coefficient in the curvature two form as $\Theta^{EF} = \frac{1}{2} R^{EF}{}_{TL} dx^T \wedge dx^L$. Thus the action of interacting part (6) can be said to have a preferred style. Note that if two metrics are identical, S_{int} becomes $-\frac{\alpha}{2} \int d^6x \sqrt{-g} F_{gMN} F_f^{MN}$.

3 Compactification of the background geometry

Compactification with the flux in the Einstein-Maxwell theory was investigated by Randjbar-Daemi, Salam and Strathdee (RSS) more than three decades

ago [20]. Therefore, existence of a similar solution for flux compactification is expected in our model.

Now, we assume that each metric describes a direct product of four-dimensional flat spacetime and an extra two sphere, S^2 . We assume that the two metrics have different scales as

$$g_{mn}dx^m dx^n = a^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad f_{mn}dx^m dx^n = b^2(d\theta^2 + \sin^2 \theta d\varphi^2), \quad (10)$$

where θ and φ are the standard coordinates on S^2 , $m, n = 5, 6$, and a and b are the radii of spheres. Then the Ricci tensors associated with two metrics are independently given by

$$R_{gmn} = \frac{1}{a^2}g_{mn}, \quad R_{fmn} = \frac{1}{b^2}f_{mn}, \quad (11)$$

where R_{mn} denotes the Ricci tensor.

Then, we suppose that the constant ‘magnetic’ flux penetrates the extra sphere, just as in the RSS model [20, 21]. Namely, we set

$$F_g = dA_g = -\frac{n_g}{2ea^2}a d\theta \wedge a \sin \theta d\varphi \quad (12)$$

and

$$F_f = dA_f = -\frac{n_f}{2eb^2}b d\theta \wedge b \sin \theta d\varphi. \quad (13)$$

Here the electric charge e has been chosen a common value, for simplicity.

We seek the background solution with the four-dimensional flat spacetime. According to work of Wetterich [22], we use the method of the effective potential for a static solution, instead of solving the equation of motion derived from the action directly. We now define a potential corresponding to the action and ansätze as

$$\begin{aligned} V(a, b) &= a^2 \left(-\frac{1}{\kappa_g^2 a^2} + \frac{n_g^2}{8e^2 a^4} + \Lambda_g \right) + b^2 \left(-\frac{1}{\kappa_f^2 b^2} + \frac{n_f^2}{8e^2 b^4} + \Lambda_f \right) \\ &\quad + 2\alpha ab \left(\frac{n_g n_f}{8e^2 a^2 b^2} \right). \end{aligned} \quad (14)$$

If we take new quantities $y \equiv ab$ and $x \equiv b/a$, the potential takes the form:

$$V(x, y) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{1}{8e^2 y} \left(n_g^2 x + \frac{n_f^2}{x} + 2\alpha n_g n_f \right) + y \left(\frac{\Lambda_g}{x} + \Lambda_f x \right). \quad (15)$$

Then, the equations of motion are expected to be satisfied if [22]

$$\left. \frac{\partial V}{\partial x} \right|_{x=x_0, y=y_0} = \left. \frac{\partial V}{\partial y} \right|_{x=x_0, y=y_0} = 0 \quad (16)$$

and

$$V(x_0, y_0) = 0. \quad (17)$$

The equation (17) requires a vanishing four-dimensional cosmological constant. To make the equations (16, 17) simultaneously satisfied, we must tune the values of κ_g and κ_f to be specific values.

Since the last two terms in (15) is positive, the minimum value of V when the value of y moves turns out to be

$$V(x, y_0) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{1}{\sqrt{2}e} \sqrt{\left(n_g^2 x + \frac{n_f^2}{x} + 2\alpha n_g n_f\right) \left(\frac{\Lambda_g}{x} + \Lambda_f x\right)}, \quad (18)$$

with

$$y_0 = \frac{1}{2\sqrt{2}e} \sqrt{\left(n_g^2 x + \frac{n_f^2}{x} + 2\alpha n_g n_f\right) \left(\frac{\Lambda_g}{x} + \Lambda_f x\right)^{-1}}, \quad (19)$$

In a special case for $\Lambda_g = n_g^{-2}\lambda$ and $\Lambda_f = n_f^{-2}\lambda$, the minimum of $V(x, y_0)$ is achieved at $x_0 = \left|\frac{n_f}{n_g}\right|$ and it takes the value

$$V(x_0, y_0) = -\frac{1}{\kappa_g^2} - \frac{1}{\kappa_f^2} + \frac{\sqrt{2\lambda}}{e} \sqrt{1 + \alpha \text{sign}(n_g n_f)}, \quad (20)$$

and then $y_0 = \frac{|n_g n_f|}{2e} \sqrt{\frac{1 + \alpha \text{sign}(n_g n_f)}{2\lambda}}$. Finally, we tune the constants as

$$\frac{1}{\kappa_g^2} + \frac{1}{\kappa_f^2} = \frac{\sqrt{2\lambda}}{e} \sqrt{1 + \alpha \text{sign}(n_g n_f)}. \quad (21)$$

In the present naive approach, the geometry of background is determined, while the individual values for κ_g and κ_f cannot be specified. The stability condition for flat four-dimensional spacetime is needed for this determination. We will see the condition in the subsequent section.

Taking further simplification, such that $\Lambda_g = \Lambda_f \equiv \Lambda$, $\kappa_g = \kappa_f \equiv \kappa$ and $n_g = n_f \equiv n$, we find $x_0 = 1$ and $y_0 = \frac{n}{2e} \sqrt{\frac{1+\alpha}{2\Lambda}}$, or equivalently, $a^2 = b^2 = \frac{n}{2e} \sqrt{\frac{1+\alpha}{2\Lambda}}$, $\frac{1}{\kappa^2} = \frac{n\sqrt{\Lambda/2}}{e} \sqrt{1+\alpha}$.

4 The masses of gravitons

In this section, we consider the dynamical graviton modes of the lowest excitation on the background. Here we do not discuss on the Kaluza-Klein excited modes.

Provided that the background geometry is one obtained in the previous section, the four-dimensional action for gravitons can be written by

$$S^{(4)} = 4\pi \int d^4x \left\{ \sqrt{-g^{(4)}} \left[\frac{a^2}{2\kappa_g^2} R_g^{(4)} + \frac{1}{\kappa_g^2} - \frac{n_g^2}{8e^2 a^2} - \Lambda_g a^2 \right] \right\}$$

$$\begin{aligned}
& + \sqrt{-f^{(4)}} \left[\frac{b^2}{2\kappa_g^2} R_f^{(4)} + \frac{1}{\kappa_f^2} - \frac{n_f^2}{8e^2 b^2} - \Lambda_f b^2 \right] \\
& - \frac{\alpha}{12} \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} e_{g\mu}^a e_{f\nu}^b e_{g\rho}^c e_{f\sigma}^d \frac{n_g n_f}{8e^2 ab} \Big\} , \tag{22}
\end{aligned}$$

where $\mu, \nu = 0, 1, 2, 3$ and $a, b = 0, 1, 2, 3$ and the superscript ‘(4)’ indicates the four dimensional quantities constructed from four dimensional metrics. Hereafter, e_μ^a stands for vierbeins.

In the weak field limit [15, 16, 17], *i.e.* $e_g = \eta + \frac{1}{2}h_g$, $e_f = \eta + \frac{1}{2}h_f$, we find

$$\sqrt{-g^{(4)}} = \det e_g = 1 + \frac{1}{2}[h_g] + \frac{1}{8}[h_g]^2 - \frac{1}{8}[h_g^2] + O(h^3), \tag{23}$$

with a similar expression for $\sqrt{-f^{(4)}}$, and

$$\begin{aligned}
& \frac{1}{24} \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} e_{g\mu}^a e_{f\nu}^b e_{g\rho}^c e_{f\sigma}^d \\
& = 1 + \frac{1}{4}([h_g] + [h_f]) + \frac{1}{48}([h_g]^2 + 4[h_g][h_f] + [h_f]^2) \\
& \quad - \frac{1}{8}([h_g^2] + 4[h_g h_f] + [h_f^2]) + O(h^3). \tag{24}
\end{aligned}$$

Here η denotes the four dimensional flat metric, and $[A] \equiv \text{tr } A$ for notational simplicity. It is known that the asymmetric part of h can be omitted [17].

Now we can write down non-derivative terms in the four-dimensional action. The constant term disappears due to Eq. (17) for the background metrics obtained in the previous section. The appearance of the linear term in $[h_g]$, $[h_f]$ bring about the instability of the flat four-dimensional spacetime. The vanishing of coefficients of linear terms in the four-dimensional action tells us

$$\frac{1}{\kappa_g^2} = \frac{n_g^2 x_0}{8e^2 y_0} + \frac{\alpha n_g n_f}{8e^2 y_0} + \Lambda_g \frac{y_0}{x_0}, \tag{25}$$

$$\frac{1}{\kappa_f^2} = \frac{n_f^2}{8e^2 x_0 y_0} + \frac{\alpha n_g n_f}{8e^2 y_0} + \Lambda_f x_0 y_0. \tag{26}$$

Since Eq. (17) still holds, the sums of each hand side of two equations (25) and (26) are equal and the values for κ_g and κ_f can be obtained consistently. Incidentally, for the case of the RSS model, the single condition of vanishing cosmological constant in four dimensions (17) implies the stability of the flat four-dimensional spacetime, because there is only one gravitational field. Actually, Eqs. (25, 26) turn out to be just the constraints which come from the variations of two lapse functions e_{g0}^0 and e_{f0}^0 .

When all the equations on the metrics including (25) and (26) are satisfied, non-derivative term in the four-dimensional action becomes very simple as

$$\frac{\alpha n_g n_f}{96e^2 y_0} ([h_g] - [h_f])^2 - [(h_g - h_f)^2]. \tag{27}$$

On the other hand, the kinetic terms for graviton fields come from

$$\int d^4x \sqrt{-g^{(4)}} R_g^{(4)} = \int d^4x \left[-\frac{1}{4} \partial_\rho h_{g\mu\nu} \partial^\rho h_g^{\mu\nu} + \frac{1}{2} \partial_\rho h_g^\rho{}_\mu \partial_\nu h_g^{\nu\mu} - \frac{1}{2} \partial_\mu h_g^{\mu\nu} \partial_\nu h_g + \frac{1}{4} \partial_\rho h_g \partial^\rho h_g + O(h^3) \right] \quad (28)$$

and a similar expression for f , where $h \equiv [h]$ for simplicity.

Therefore the Lagrangian for linearized graviton fields is written by

$$\begin{aligned} & \frac{y_0}{2\kappa_g^2 x_0} \left[-\frac{1}{4} \partial_\rho h_{g\mu\nu} \partial^\rho h_g^{\mu\nu} + \frac{1}{2} \partial_\rho h_g^\rho{}_\mu \partial_\nu h_g^{\nu\mu} - \frac{1}{2} \partial_\mu h_g^{\mu\nu} \partial_\nu h_g + \frac{1}{4} \partial_\rho h_g \partial^\rho h_g \right] \\ & + \frac{x_0 y_0}{2\kappa_f^2} \left[-\frac{1}{4} \partial_\rho h_{f\mu\nu} \partial^\rho h_f^{\mu\nu} + \frac{1}{2} \partial_\rho h_f^\rho{}_\mu \partial_\nu h_f^{\nu\mu} - \frac{1}{2} \partial_\mu h_f^{\mu\nu} \partial_\nu h_f + \frac{1}{4} \partial_\rho h_f \partial^\rho h_f \right] \\ & + \frac{\alpha n_g n_f}{96e^2 y_0} \left[(h_g - h_f)^2 - (h_{g\mu\nu} - h_{f\mu\nu})^2 \right] \\ & = -\frac{1}{2} \partial_\rho H_{0\mu\nu} \partial^\rho H_0^{\mu\nu} + \partial_\rho H_0^\rho{}_\mu \partial_\nu H_0^{\nu\mu} - \partial_\mu H_0^{\mu\nu} \partial_\nu H_0 + \frac{1}{2} \partial_\rho H_0 \partial^\rho H_0 \\ & - \frac{1}{2} \partial_\rho H_{1\mu\nu} \partial^\rho H_1^{\mu\nu} + \partial_\lambda H_1^\lambda{}_\mu \partial_\nu H_1^{\nu\mu} - \partial_\mu H_1^{\mu\nu} \partial_\nu H_1 + \frac{1}{2} \partial_\rho H_1 \partial^\rho H_1 \\ & + \frac{1}{2} \frac{\alpha n_g n_f}{12e^2 y_0^2} \left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0} \right) (H_1^2 - H_{1\mu\nu}^2), \end{aligned} \quad (29)$$

where

$$H_0 \equiv \frac{\frac{\kappa_f}{\kappa_g x_0} h_g + \frac{\kappa_g x_0}{\kappa_f} h_f}{\sqrt{\frac{4}{y_0} \left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0} \right)}}, \quad H_1 \equiv \frac{h_g - h_f}{\sqrt{\frac{4}{y_0} \left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0} \right)}}. \quad (30)$$

The quadratic term of H_1 corresponds to the Fierz-Pauli mass term [23]. Therefore we conclude that the present model with the previously-obtained background geometry contains one massless graviton field H_0 and one massive graviton field H_1 . The mass squared m^2 of H_1 is given by

$$m^2 = \frac{\alpha n_g n_f}{12e^2 y_0^2} \left(\kappa_g^2 x_0 + \frac{\kappa_f^2}{x_0} \right), \quad (31)$$

if $\alpha n_g n_f$ is positive.

Now we examine the simple cases seen already in the last part of the previous section. In the special case for $\Lambda_g = n_g^{-2} \lambda$ and $\Lambda_f = n_f^{-2} \lambda$, we found $x_0 = \left| \frac{n_f}{n_g} \right|$

and $y_0 = \frac{|n_g n_f|}{2e} \sqrt{\frac{1+|\alpha|}{2\lambda}}$. The gravitational constants should be chosen as

$$\frac{1}{\kappa_g^2} = \frac{1}{\kappa_f^2} = \frac{\sqrt{\lambda/2}}{e} \sqrt{1+|\alpha|}. \quad (32)$$

In this case, we find that the mass of the massive graviton is

$$m^2 = \frac{2\sqrt{2\lambda}\alpha e}{3(1+|\alpha|)^{3/2}} \frac{n_g^2 + n_f^2}{n_g^2 n_f^2}. \quad (33)$$

In the further simple case for $\Lambda_g = \Lambda_f \equiv \Lambda$, $\kappa_g = \kappa_f \equiv \kappa$ and $n_g = n_f \equiv n$, we found $x_0 = 1$ and $y_0 = \frac{n}{2e} \sqrt{\frac{1+\alpha}{2\Lambda}}$, $\frac{1}{\kappa^2} = \frac{n\sqrt{\Lambda/2}}{e} \sqrt{1+\alpha}$, and $0 < \alpha < 1$. The mass of the massive graviton is

$$m^2 = \frac{4\sqrt{2\Lambda}\alpha e}{3n(1+\alpha)^{3/2}}. \quad (34)$$

Note that since the ratio $(m^2/\frac{2}{a^2}) = \frac{\alpha}{3(1+\alpha)}$ is always smaller than one, the massive graviton is expected to be lighter than the first Kaluza-Klein excited mode of massless graviton.

5 Summary and outlook

In the present paper, we presented a model of bimetric theory in six dimensions. We showed that the compactification with fluxes in the extra space leads to the four-dimensional massive and massless gravity. The relation of parameters which allows the compactification has been obtained. We found the relation in the mass of the massive graviton and the parameters in the model.

It is interesting to see that two radii of the extra spheres in two metrics can have different values in general. This fact will be of more importance if we consider the Kaluza-Klein towers of excitation of gravitons as well as matter fields. The possible variety of mass spectrum is worth studying in some phenomenological and cosmological contexts. It is interesting to study the model as a quantum field theory, because of complexity of the interaction among the infinite Kaluza-Klein excited modes of gravitons, gauge fields, and matter fields to be added in the ‘asymmetric compactification’ for $a \neq b$. It is also interesting to introduce a dilaton fields into bimetric models as in six dimensional supergravity model [24, 25]. We think that similar compactifications can be considered in such models. However, we suspect that there are possibilities of removing the tuning of couplings and even to eliminate the asymmetry of two spheres in models with a dilatonic field.

The cosmological application of our model attracts much attention, for massive gravity is expected to solve the riddle of cosmic acceleration. Our simple model is suitable to study of classical and quantum cosmology, since cosmological aspects of the RSS model have been studied by Okada [26, 27], Halliwell [28, 29], and the succeeding authors.

Finally, we notify that it is straightforward to generalize the present model to the model of multigravity [18, 30]. In such a theory, one may expect to find a new hierarchical spectrum of gravitons and the other fields in the theory. It is interesting to study such a model, because complicated particle content may cause interesting quantum effects as well as novel cosmological evolution.

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